NON-NEWTONIAN GROUT FLOW IN A SINGLE ROUGH-WALLED ROCK FRACTURE

ICKE-NEWTONSK STRÖMNING AV INJEKTERINGSBRUK I ENSKILDA RÅA SPRICKOR

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Abstract
Modeling of cement grout flow in rock fractures plays an important role in the design of rock grouting. Cement grouts used in rock grouting practice are typical non-Newtonian fluids containing yield stress, and are often assumed as Bingham fluids. Natural rock fractures typically consist of rough surfaces. Therefore, in reality, rock grouting process actually involves non-Newtonian fluid flow in rough-walled fractures, which is rarely studied in the literature. In this work, we focus on the impact of surface roughness and present direct numerical simulations of non-Newtonian grouts flow in single rough-walled fractures, using a regularized method to approximate the yield-stress. The rough-walled rock fracture models are created from a laser-scanned surface of a granite rock sample, to represent realistic features of natural rock fractures. The numerical results generally show nonlinear behaviors of non-Newtonian fluid flow in rough-walled fractures. The surface roughness significantly reduces the effective transmissivity when Reynolds number is relatively large. The obtained result can be used for upscaling analysis in practice, in order to reduce the potential uncertainties caused by the surface roughness of the rock fractures.

Sammanfattning
Modellering av cement-baserade injekteringsmedels strömning i bergsprickor spelar en viktig roll för prediktion och design. De medel som används vid injektering är typiskt icke-Newtonska, med en flytgräns, och antas därför vara av Bingham typ. Naturliga bergsprickor har vanligtvis en rå och ojämn yta. Analyser av injekteringsförlopp borde därför innehålla både icke-Newtonska vätskor och råhet hos sprickorna, vilket oftast inte är fallet i den litteratur som finns tillgänglig idag. I föreliggande arbete fokuseras på inverkan av bergsprickors råhet och numeriska beräkningar redovisas för icke-Newtonska injekteringsmedels strömning i enskilda sprickor, med hjälp av en regulariserad metod för hantering av brukets flytgräns. Sprickytornas råhet modelleras från laser-scannade ytor av granitprover för att erhålla så realistiska förhållanden som möjligt. Resultaten visar på icke-linjära effekter och att de råa sprickytorna har en avgörande inverkan på spricktransmissiviteten och att resultaten kan användas för att minska osäkerheten vid praktisk tillämpning.

1. Introduction

Modeling of cement grout flow in rock fractures is important for design and performance of grouting activities in underground rock engineering projects (Stille 2015). In practice, cement grouts are typically non-Newtonian fluids assumed as Bingham fluids

(Håkansson et al. 1992; Håkansson 1993). In the literature, many analytical and numerical models have been developed to analyze grouting in both single homogeneous planar rock fractures (e.g., Lombardi 1985; Hässler 1991; Stille et al. 2009; Stille 2015; Zou et al. 2018a), and discrete fracture networks (Hässler 1991; Eriksson et al. 2000; Zou et al. 2019). These studies advanced the theoretical development for modeling of rock grouting. However, they were all based on the important assumption that the fracture is homogeneous with a constant aperture.

In reality, rock fracture surfaces are rough and fracture apertures are spatially variable, which causes important uncertainty for modeling of fluid flow and solute transport in fractured rocks (e.g., Zou et al. 2016; 2017). For Newtonian groundwater, many experimental and numerical studies revealed that the rock fracture surface roughness significantly reduces the effective transmissivity of rock fractures and the hydraulic aperture is therefore smaller than the mechanical aperture (Zou et al. 2015; 2017). However, for non-Newtonian fluids, e.g., cement grouts and drilling muds, such impact of fracture surface roughness have not been investigated. Therefore, the consequence of the assumption that the fracture is homogeneous, with a constant aperture as adopted by many theoretical studies, has not been studied in previous works.

In this study, we aim to investigate the impact of surface roughness on non-Newtonian cement grout flow in rock fractures by direct numerical simulations of Bingham grout flow in a single rough-walled fracture, using a regularized method to approximate the yield-stress. The rough-walled rock fracture models are created from a laser-scanned surface of a granite rock sample, to represent realistic features of natural rock fractures. The results demonstrate the various flow behaviors and the obtained effective transmissivity can be used for upscaling analysis in practice, in order to reduce the potential uncertainties caused by surface roughness of rock fractures.

2. Theory and analytical solution

2.1 Governing equations for Bingham fluids flow

The general governing equations based on mass and momentum conservation for incompressible fluid flow can be written as

$$\nabla \cdot \mathrm{u} = 0 \quad (1)$$

$$\frac{\partial}{\partial \mathrm{t}}(\rho \mathrm{u}) + \rho\, \mathrm{u} \cdot \nabla \mathrm{u} = -\nabla \mathrm{p}\mathrm{I} + \nabla \cdot \tau + \rho \mathrm{g} \quad (2)$$

where u (m/s) is the fluid velocity vector, ρ is fluid density, t is time, P is pressure, I is the identity matrix and g is acceleration of gravity.

The cement grouts used for grouting in practice are typically non-Newtonian fluids (Håkansson 1993). The linear Bingham rheology model was widely adopted for theoretical analyses in the past decades, given by

$$\begin{cases} \tau_{ij} = \left(\frac{\tau_0}{|\dot{\gamma}_{ij}|} + \mu_B\right) \dot{\gamma}_{ij}, & |\tau_{ij}| > \tau_0 \\ \dot{\gamma}_{ij} = 0 & \text{otherwise} \end{cases} \quad (3)$$

where τ_{ij} is shear stress tensor, $\dot{\gamma}_{ij}$ is the shear rate (rate-of-strain) tensor, τ_0 is the yield stress and μ_B is the plastic viscosity. The Bingham fluid is an idealized non-Newtonian fluid that may not exist in reality. Papanastasiou (1987) developed a modified Bingham model (referred to as the Bingham-Papanastasiou model) by adding an exponential term to smoothly approximate the yield stress, which is able to represent the more realistic visco-plastic behavior of non-Newtonian fluids, written as

$$\tau_{ij} = \left\{\mu_B + \frac{\tau_0}{|\dot{\gamma}_{ij}|}\left[1 - \exp(-m|\dot{\gamma}_{ij}|)\right]\right\} \dot{\gamma}_{ij} \tag{4}$$

where the m is an exponential index. As m is relatively large, i.e., m = 100, the Bingham-Papanastasiou model approaches the Bingham model. The Bingham-Papanastasiou model is valid for all regions, both yielded and un-yielded. Therefore, it avoids determining the yield surface, which brings important advantages especially for numerical modeling.

2.2 Unidirectional flow of Bingham fluid in homogeneous fractures

According to these assumptions, the governing equations for the single-phase of a Bingham fluid flow in a homogenous fracture can be simplified as,

$$\frac{\partial u}{\partial x} = 0 \tag{5}$$

$$-\frac{\partial P}{\partial x} = \frac{\partial \tau}{\partial y} \tag{6}$$

For Bingham fluid flow in a homogenous fracture with a given pressure gradient, the simplified governing equations can be analytically solved. The pressure gradient and velocity are given by

$$\frac{\partial P}{\partial x} = \frac{P_2 - P_1}{L} \tag{7}$$

$$u(y) = \frac{1}{2\mu_B}\frac{dP}{dx}(B^2 - y^2) + \frac{\tau_0}{\mu_B}(B - y) \quad z_p < z \leq B \tag{8}$$

$$u(y) = \frac{1}{2\mu_B}\frac{dP}{dx}\left(B^2 - {z_p}^2\right) + \frac{\tau_0}{\mu_B}\left(B - z_p\right) \quad 0 < z \leq z_p \tag{9}$$

where B is half of the aperture, L is the fracture length and z_p is half of the plug flow region caused by the yield stress, determined by

$$z_p = \min\left(\frac{\tau_0 L}{P_1 - P_2}, B\right) \tag{10}$$

3. Numerical simulations

3.1 Numerical solution method and simulation scenarios

We extended our in-house code FracFlow to directly simulate the Bingham grouts flow in a single rock fracture. The FracFlow code is an unstructured Finite Volume Method (FVM) code developed to directly solve fluid flow and solute transport in single rock fractures. The detailed numerical schemes can be found in Zou et al. (2015; 2016). A single rough-walled fracture model has been created from a laser-scanned digital surface of a granite rock sample. The detailed statistics of the digital surface and this created fracture model can be found in Zou et al. (2014; 2015). For all simulations, the inlet boundaries are set with constant flow rate, and the outlet boundaries are set with zero pressure. The upper and lower plates are set as no-slip walls, i.e. with zero velocity. The parameters adopted for the numerical simulation are summarized in Table 1.

Table 1 Geometrical and physical parameters adopted for the numerical simulation.

Parameters	Units	Values
Fracture length, L	[m]	0.1
Mean aperture, $2B$	[m]	1e-3
Viscosity of grout, μ_g	[Pa·s]	0.025
Yield stress of grout, τ_0	[Pa]	5
Density of grout, ρ_g	[kg/m^3]	1500
Reynolds number, $Re = 2Bu\rho_g/\mu_g$	[-]	0.05, 0.1, 0.5, 1, 5, 10, 50, 100, 500, 1000

3.2 Velocity field

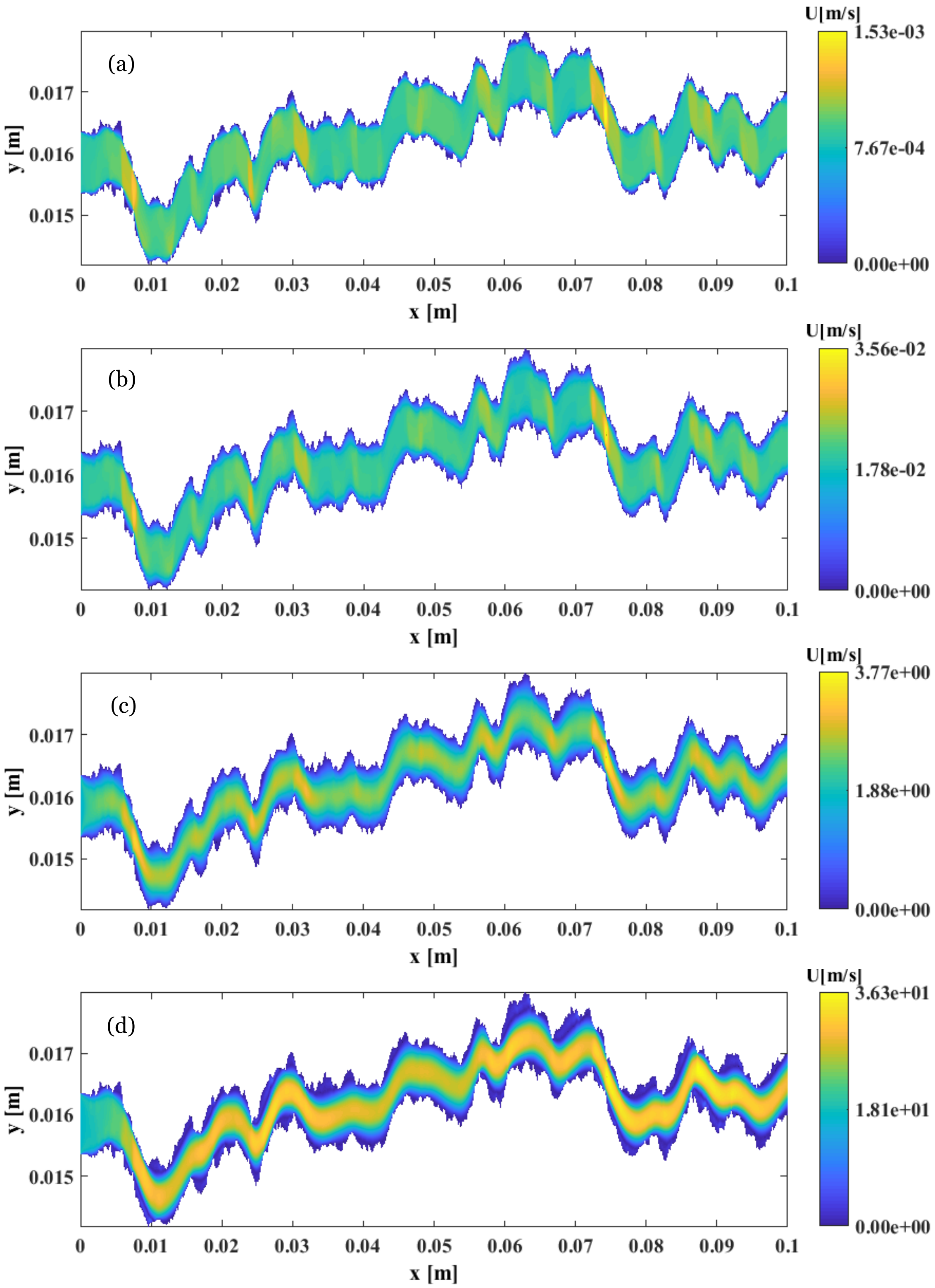

Figure 1 Overview of velocity distribution in the single rough-walled fracture, (a) Re = 0.05, (b) Re = 1, (c) Re = 100 and (d) Re = 1000.

Figure 1 presents representative results of the overall velocity field distributions with Re = 0.05, 1, 100 and 1000, where the contour maps represent the magnitude of the velocity field, i.e. $\sqrt{u^2+v^2}$. When the Re is relatively small, i.e. Re ≤ 1, the high velocity regions are distributed discretely along the fracture where the aperture is relatively small. In contrast, the low velocity regions are distributed near the rough walls where the aperture is relative large including sharp cornered asperities of the wall surfaces. When the Re is relatively high Re $\geq$ 100, the high velocity regions are concentrated in the middle of aperture and the relatively low velocity regions become larger than the cases with smaller Re.

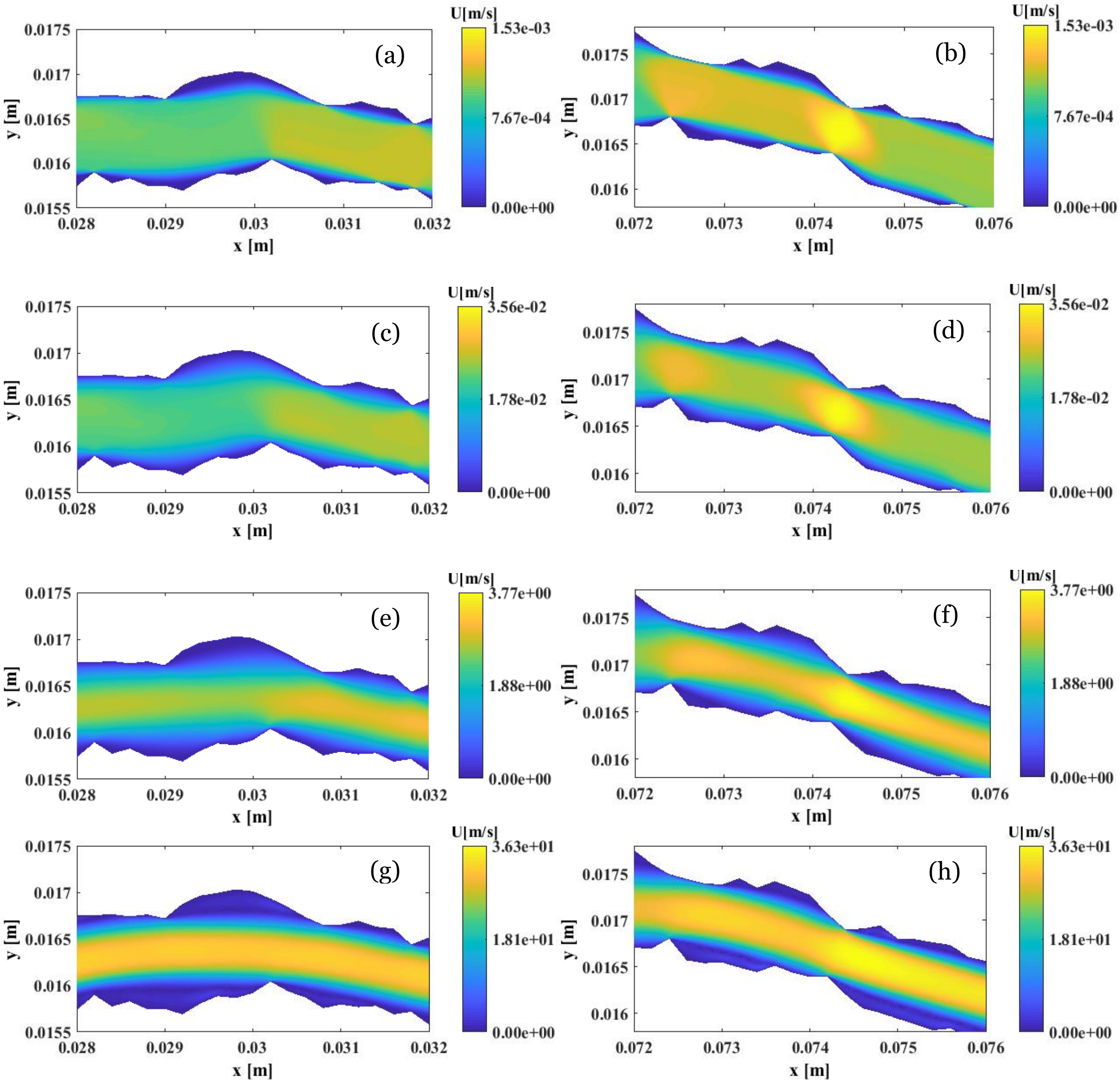

Figure 2 Velocity field in the sections between x = [0.028m, 0.032m] and x = [0.072m, 0.076m] with different Re values. (a) x = [0.028m, 0.032m], Re = 0.05; (b) x = [0.072m, 0.076m], Re = 0.05; (c) x = [0.028m, 0.032m], Re = 1; (d) x = [0.072m, 0.076m], Re = 1; (e) x = [0.028m, 0.032m], Re = 100; (f) x = [0.072m, 0.076m], Re = 100; (g) x = [0.028m, 0.032m], Re = 1000; (h) x = [0.072m, 0.076m], Re = 1000.

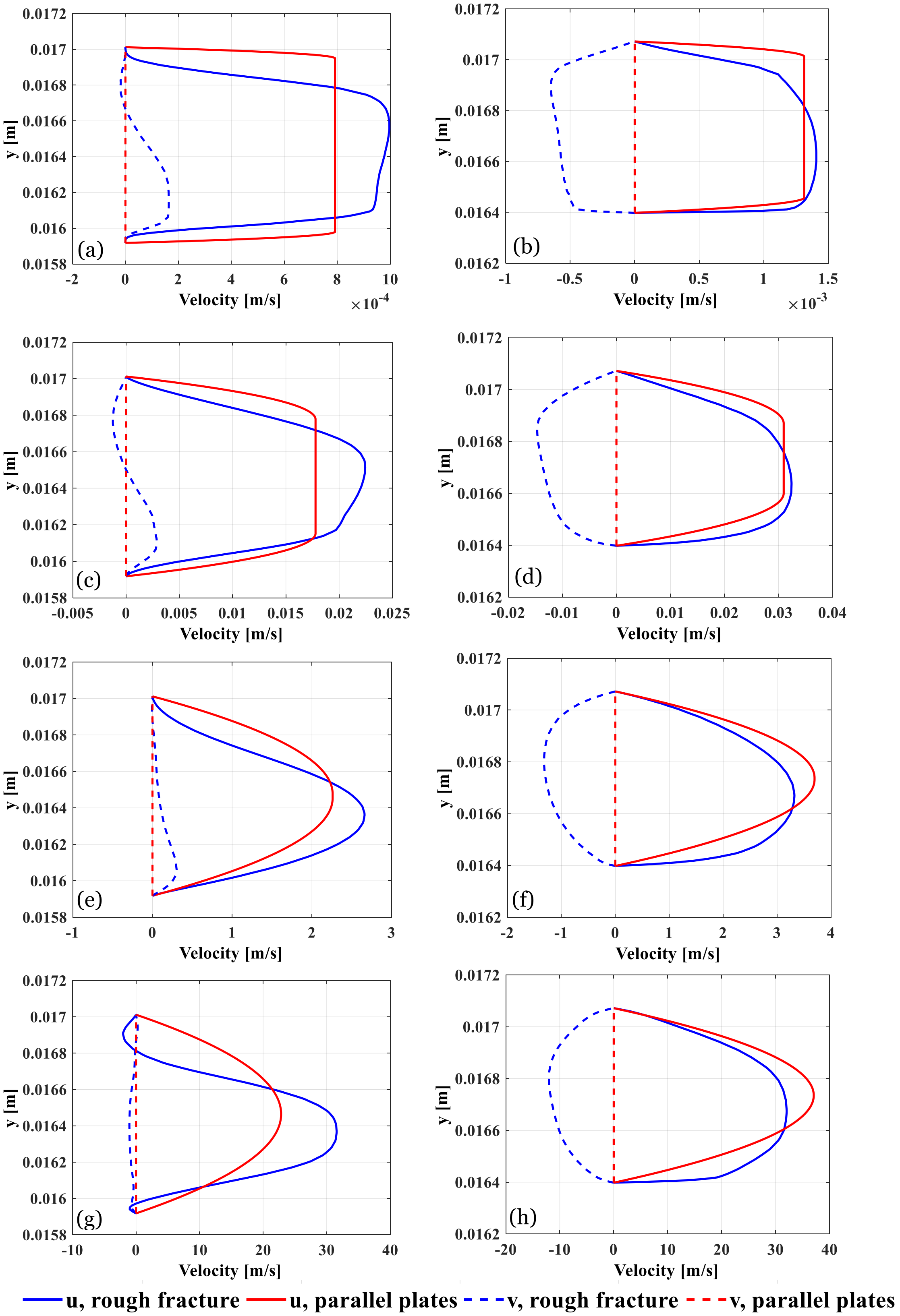

Figure 3 Local velocity profiles in the sections between x = 0.032m and x = 0.074m with different Re values: (a) x = 0.032m, Re = 0.05; (b) x = 0.074m, Re = 0.05; (c) x = 0.032m, Re = 1; (d) x = 0.074m, Re = 1; (e) x = 0.032m, Re = 100; (f) x = 0.074m, Re = 100; (g) x = 0.032m, Re = 1000; (h) x = 0.074m, Re = 1000. The u is the velocity on the x-axis direction and the v is the velocity on the y-axis direction.

To show the detailed flow regions in the rough-walled fracture, two typical cross-sections contain a relatively large aperture zone and a relatively small aperture zone respectively, are presented in Figure 2. For cases Re ≤ 1, the relatively high flow regions in the middle part of the fracture exhibits pseudo plug flow behaviors caused by the yield stress. Such pseudo plug flow behavior for non-Newtonian grouts is very different from the Newtonian groundwater. These pseudo plug flow regions are mostly affected by the small aperture zones and less affected by the large aperture zones caused by the surface asperities. With Re increases, the high velocity flow region becomes much narrower caused by the increasing inertial effect.

In order to show the local flow behavior in the rough-walled fracture, the velocity profiles of cross-sections at the location of x=0.03 m and x=0.074 m, with Re numbers of 0.05, 1, 100 and 1000, are presented in Figure 3. In addition, the velocity profiles calculated by using the analytical solution for the idealized smoothed parallel plates model through Eq. (8) and (9) are also presented for comparison. The velocity profiles for the rough-walled fracture by numerical simulation are generally irregular and as expected cannot match with the idealized profiles by the analytical solution for homogeneous fractures.

In particular, when Re = 1000, the velocity profiles at the location of x=0.03 m show negative values near the rough walls, indicating there are eddy flows near the rough walls. Similar to Newtonian groundwater flow in rough-walled fractures, such eddy flows are caused by the fracture surface roughness and inertial effects. Note that the Re number can be very high at the initial stage of rock grouting due to high pressure gradients by using the constant injection pressure method. Therefore, such eddy flows or even turbulent flows may certainly occur in practice.

3.2 Equivalent transmissivity

Figure 4 presents the relationship between the flowrate and the overall pressure gradient for Bingham grout flow in the rough-walled fracture. The theoretical relationship between the flowrate and pressure gradient for the idealized smoothed parallel plates model with the same mean aperture (1mm) is also plotted for comparison (the red curve). The flowrate for the rough-walled fracture by numerical simulation matches well with the theoretical values for the idealized homogeneous fracture when the pressure gradient is relatively small (i.e., Re ≤ 10). In contrast, the theoretical solution for idealized homogeneous fracture significantly overestimates the flowrate for the rough-walled fracture when the pressure gradient is relatively large (i.e., Re ≥ 50).

Figure 5 presents the equivalent transmissivity for different Re. The equivalent transmissivity is calculated based on the relationship between the flowrate and overall pressure gradient shown in Figure 4. The theoretical equivalent transmissivity for the idealized homogeneous fracture with the same mean aperture (1mm) is also presented for comparison (the red curve). Obviously, the equivalent transmissivity is a nonlinear function of Re, where the theoretical transmissivity increases with increasing Re until Re ≥ 50 because the yield stress caused plug flows become less important with increasing of Re. For cases with Re ≤ 10, the equivalent transmissivity for the rough-walled fracture matches well with that for the idealized homogeneous fracture. It indicates that the fracture surface roughness has limited impact on the equivalent transmissivity when Re is relatively small, where the plug flow dominates the equivalent transmissivity. However, for cases of Re ≥ 50, the equivalent transmissivity for the idealized homogeneous fracture approaches a constant value whereas the equivalent transmissivity for the rough-walled fracture decreases with increasing Re, due to the increasing inertial effects. This result indicates that the surface roughness only affect the equivalent transmissivity when Re is relatively large, i.e. Re ≥ 50.

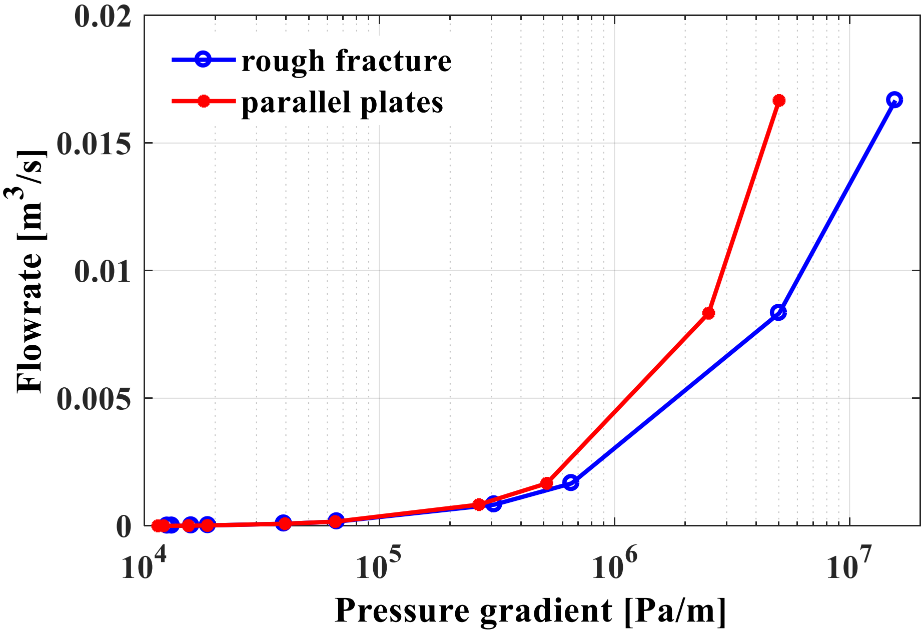

Figure 4 Relationship between the flowrate and overall pressure gradient.

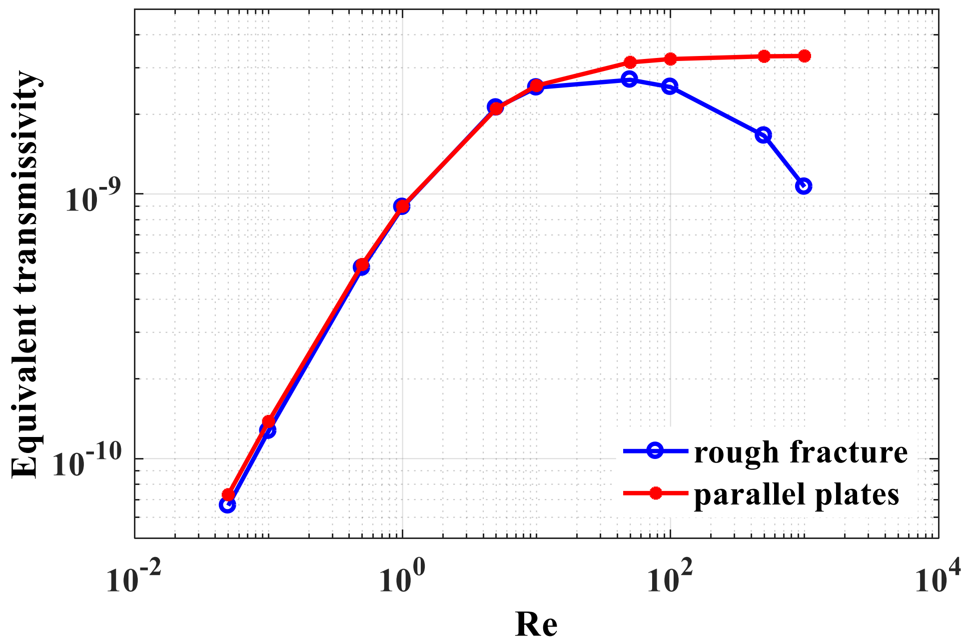

Figure 5 Equivalent transmissivity for different Re.

4. Discussion

For Newtonian fluids, i.e. groundwater, the relationship between flowrate and pressure gradient is theoretically linear and the transmissivity is independent on the pressure gradient for homogeneous fractures under laminar flow conditions. In contrast, the relationship between flowrate and pressure gradient for non-Newtonian grouts is naturally nonlinear and the equivalent transmissivity is dependent on the pressure gradient because of the yield stress.

Previous studies for groundwater flow in rough-walled fractures have shown that the fracture surface roughness significantly affect the equivalent transmissivity, even when Re is relatively small, i.e., Re < 0.1 (e.g., Zou et al. 2015). However, the present results show that the fracture surface roughness has limited effect on the equivalent transmissivity when Re ≤ 10, indicating that the theoretical transmissivity based on the analytical solution for idealized homogeneous fracture is valid for rough-walled fractures when Re is relatively small, i.e., Re ≤ 10. When Re is relatively large, i.e. Re > 10, the equivalent transmissivity for the non-Newtonian grout flow in rough-walled fractures reduces dramatically, which is similar to the Newtonian groundwater flow in rough-walled fractures because of the roughness and inertial effects (e.g., Zou et al. 2014; 2015). Therefore, ignoring the impact of surface roughness for the cases with relatively large Re conditions may overestimate the effective transmissivity for modeling of non-Newtonian fluid flow in natural rock fractures.

In rock grouting practice, non-Newtonian grouts are often injected with a constant pressure (Stille 2015). At the initial stage of injection, the Reynolds number can be very large when the surface roughness and inertial effects are significant. Therefore, penetration models based on idealized homogeneous fracture may overestimate the penetration length of the grouts, and need to be quantified properly for practical applications.

5. Conclusions

In this study, non-Newtonian grout flow behavior in a single rough-walled rock fracture are directly simulated and the impact of fracture surface roughness on the equivalent transmissivity is investigated. Some important conclusions obtained from this study are summarized below:

- The fracture surface roughness significantly affect the local flow behavior for Bingham grout flow in rough-walled rock fractures. The local velocity profiles cannot be well predicted by the analytical solution based on an idealized smoothed parallel plate model.
- When the Reynolds number is relatively small, i.e., Re $\leq$ 10, the surface roughness has limited impact on the overall flow behavior and the equivalent transmissivity for the rough-walled fracture can be well approximated by the analytical solution without consideration of the surface roughness.
- When the Reynold number is relatively large, i.e., Re > 10, the equivalent transmissivity for the rough-walled fracture reduces with increasing Re. Using the analytical solution will overestimate the more realistic transmissivity in such conditions.
- For rock grouting design, ignoring the impact of fracture surface roughness may overestimate the penetration length, especially at the initial stage of injection where the Reynolds number is high.

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